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Flexural Analysis of RC Beams Strengthened with Externally Bonded FRP Sheets

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Abstract: Modern construction industries are very much keen in using fibre reinforced polymer (FRP) composites in retrofitting the deteriorated concrete structures for enhancing its strength and performance. Though there are several national guidelines on flexural strengthening of concrete structures using externally bonded FRP laminates/sheets, the design guidelines are yet to be recommended in India. In order to fulfil the above objective, an analytical model of RC beam retrofitted with externally bonded FRP is developed in compatible with IS:456-2000. Thereafter, a computer code is developed employing the above mathematical formulation. Finally, design charts have been prepared for the strengthened RC beams retrofitted with FRP in terms of non-dimensional parameters so that the designers can use the charts for obtaining different parameters, such as thickness of FRP, area of tensile steel, etc as in the case of SP-16-1978 or RC beams without FRP.

Keywords: Design-aids, RC beams, FRP, Retrofitting, Flexural strength

I. Introduction

Deterioration in concrete structures is a major challenge faced by the infrastructure and bridge industries worldwide. The deterioration is mainly due to environmental effects, which includes corrosion of steel, variation in temperature, freeze-thaw cycles, contact with chemicals and saline water and exposure to ultra-violet radiations. This fact leads to the necessity for development of appropriate structural retrofit/repair systems. Moreover, strengthening measures are required in structures when they are required to accommodate increased loads. Also, when there are changes in the use of structures, individual supports and walls may need to be removed. This leads to a redistribution of forces and the need for local reinforcement. Errors during construction phase also demands strengthening of structure before usages.

Design and construction communities throughout the world are increasingly using fiber-reinforced polymer (FRP), mainly carbon fibre reinforced polymer (CFRP) and glass fibre reinforced polymer (GFRP) in repairing and rehabilitating the existing infrastructures. Fiber reinforced polymers (the composite materials) are ideally suited for repair and strengthening of concrete structures in aggressive environments due to their non-corrosive, non-magnetic characteristics. They have high tensile strength to weight ratio and high elastic limit. Externally applied FRP sheets or laminates are bonded directly to a concrete surface with an epoxy providing additional flexural or shear strength capacity depending on the application and fiber alignment. This significantly increases the load carrying capacity of a structural component and/or structural system. The next section presents a brief review in this field.

II. Brief Review of Literature

A lot of research works have been done and some field applications have also been conducted on the strengthening of R. C. beams/girders using externally bonded FRP laminates/sheets. Several investigators carried out experimental and/or theoretical investigations on concrete beams/girders retrofitted with carbon/glass fibre reinforced polymer (CFRP/GFRP) composites in order to study their effectiveness [1-9]. Experimental investigations, theoretical calculations and numerical simulations showed that strengthening the reinforced concrete beams with externally bonded CFRP sheets in the tension zone considerably increased the strength at bending, reduced deflections as well as crack width [10-14]. It also changed the behaviour of these beams under load and failure pattern. Most often the strengthened beams failed in a brittle way, mainly due to the loss of contact between the composite material and the concrete. Swain and Nayak [11] presented an experimental investigation to study the behavior of RC beams retrofitted with different layers and laying patterns of externally bonded GFRP sheets and to obtain the trend of their flexural strength enhancement and failure modes.

Advanced knowledge on FRP materials and its application has been exchanged worldwide through international conferences/workshops on FRP and design guidelines in several countries have been brought forth in these days for flexural strengthening/retrofitting of concrete structures using externally bonded FRP laminates/sheets. Few of them, which are followed worldwide, are discussed in the next section. On the contrary, there exists a divergence of opinion about certain aspects of these design and detailing guidelines. This is to be expected as the use of the relatively new material develops worldwide. Much research is being carried out at institutions around the world and it is expected that design criteria will continue to be enhanced as the results of this research become known in the coming years.

III. DESIGN GUIDELINES ON FLEXURAL STRENGTHENING

There are several national guidelines on flexural strengthening of concrete structures using externally bonded FRP laminates/sheets such as ACI440:2002, fib:2000, JSCE:2001, The Concrete Society, U.K:2004, ISIS Canada:2001 [16-20]. Analytical approaches to evaluate the contribution of FRP laminates to concrete structures in flexural behavior are more or less identical in all national design guidelines. All these guidelines adopt the traditional sectional analysis called "plane sections before bending remain plane after bending" for strain compatibility, and the stress-strain relationships of concrete, steel and FRP laminates are used for equilibrium equations. With considerations of failure modes of members, a simplified method, such as a usage of stress block of concrete, is applicable. There are, however, some differences in these guidelines concerning with safety factors and necessary considerations in design. Few commonly adopted national guidelines are discussed briefly in the following sub-sections.

A. ACI-440:2002 [16]

Three safety factors are introduced for evaluation of the flexural capacity of members with FRP laminates, such as, an environmental reduction factor (C_E) for material strength for different exposure condition, a partial reduction factor (Ψ=0.85) for efficiency of FRP laminates and a strength reduction factor (Φ) to account for loss of ductility due to FRP laminate. The above guidelines also specifies the stress limit of FRP laminate for a sustain load and cyclic load.

B. fib:2001[17]

In fib: 2001 design guidelines, three safety factors are also considered. safety factors for laminate (γ_f) is taken from 1.2 to 1.5 depending upon the systems (prefab/wet lay-up systems) and types (CFRP/AFRP/GFRP) of FRP laminates. Other two safety factors are 1.5 for concrete and 1.15 for steel reinforcement. The most important safety concept for ultimate limit state is to guarantee the yielding of internal steel reinforcement so that the strengthen member will fail in a ductile manner.

C. JSCE:2001[18]

As for safety factors, JSCE: 2001 adopts five safety factors such as material safety factor (γ_m), a member factor (γ_b), a structure factor (an importance factor) (γ_i), a load factor (γ_f) and an analysis factor (γ_a).The maximum design stress is also limited in consideration of peeling failure starting at cracked portion.
Though the concrete material used in India is a lion share of the total consumption of concrete in the world, the national guidelines/codal provisions on the design of concrete structures retrofitted with FRP are yet to be recommended. The above information have impressed the authors to develop an analytical model for the flexural strengthening the R. C. beams/girders retrofitting with externally bonded FRP sheets/fabrics by adopting the traditional sectional analysis in accordance with IS:456 - 2000 [21] and to provide user-friendly design charts as in SP:16-1978 [22] for RC beams for suitability of the designers.

IV. THEORETICAL FORMULATION

The section is divided into mainly two parts. In the first part the traditional sectional analysis is made with accordance with IS:456-2000 [21] in order to derive mathematical equation for finding the ultimate flexural strength. The design charts prepared employing the above mathematical formulations for the rectangular RC beams with FRP plates are presented in the second part. Thereafter, suitable design charts are proposed to facilitate the designers to select the thickness of FRP plates and other necessary parameters for the strengthening.

A. Analysis of FRP Strengthened RC Beam

This section presents the analysis of FRP strengthened RC beams, in accordance with Indian Standard Code IS:456-2000 [21], in order to derive the relevant equations and the concerned solution steps.
The following assumptions are made to carry out the analysis of FRP strengthened RC beams.

i. The strain distribution is linear throughout the full depth of beam section, i.e. plane section normal to the axis remains plane after bending.
ii. The strain compatibility is maintained at the interface of the FRP sheets and concrete and also between steel and concrete, i.e. there is no slip between FRP sheets and concrete and also between steel and concrete.
iii. The maximum compressive strain of concrete at the outermost compression fibre is taken as 0.0035 in bending.
iv. The stress-strain distribution in concrete is assumed to be parabolic, in accordance with IS: 456-2000.
v. The tensile strength of concrete is ignored.
vi. The stresses in the steel reinforcement are derived from the representive stress-strain curve for the type of steel used, in accordance with IS: 456-2000.
vii. The maximum strain in tension reinforcement in the section at the failure shall not less than $(f_y/1.15E_s)+0.002$.
viii. The modulus of Elasticity of the FRP shall be obtained from the tensile coupon test of the FRP sheet to be used as per IS specifications.

An under-reinforced section of FRP strengthened RC beam is considered for the analysis purpose. Generally, flexural structural member like beam need to be strengthen at tension side. By retrofitting with FRP the moment carrying capacity of beam is increased by some amount depending upon the thickness of fiber used and amount of reinforcement. The detail analysis is presented below.
The cross-section of RC beam strengthened with FRP, strain diagram and stress diagram of the crosssection are shown in Fig.1. From the strain diagram in Fig. 1, the following relations can be obtained:

(Eq.1) $$\frac{0.0035}{x_u} = \frac{\varepsilon_2}{(d - x_u)}$$

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(Eq.2) $$\frac{0.0035}{x_u} = \frac{\varepsilon_3}{\left(d + d' - x_u\right)}$$

(Eq.3) $$\frac{0.0035}{x_u} = \frac{\varepsilon_4}{\left(d + d' + t_f - x_u\right)}$$

Where b, d, d', t_f, x_u, ε_1, ε_2 ε_3 and ε_4 are defined in Fig.1.

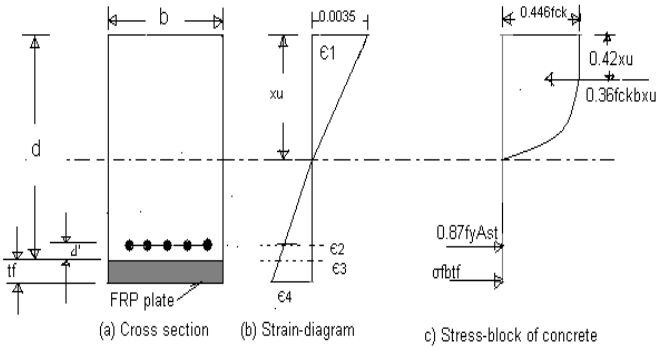

Fig. 1 Stress-strain diagrams of FRP strengthened RC beam

Strain in fiber is the average of ε_3 and ε_4 and is given by

(Eq.4) $$\varepsilon_f = \frac{\varepsilon_3 + \varepsilon_4}{2}$$

Rearranging the above equations ε_f can be obtained as

$$\varepsilon_f = \varepsilon_2 + \frac{d'}{d}\left(\varepsilon_2 + 0.0035\right)$$

(Eq.5) $$+ \frac{t_f}{2d}\left(\varepsilon_2 + 0.0035\right)$$

Now, the stress in fiber, σ_f, is given by

(Eq.6) $$\sigma_f = \varepsilon_f E_f$$

Where, E_f is the modulus of elasticity of fibre.

Now, equating total compression and tension, the depth of neutral axis can be expressed as

$$\frac{x_u}{d} = \frac{0.87 f_y p_t}{0.36 f_{ck}} + \frac{\sigma_f t_f}{0.36 f_{ck} d} \qquad \text{(Eq.7)}$$

Where, f_y and f_{ck} are yield stress in steel and Characteristic stress of concrete, respectively, and A_{st} and p_t are cross-sectional area and percentage of tensile steel reinforcement, respectively.

Ultimate moment of resistance of the FRP strengthened RC beams can be found out as follows:

$$M_u = \frac{0.87 f_y p_t b d^2}{100}\left(1 - 0.42\frac{x_u}{d}\right)$$
$$+ \frac{t_f}{d} b d^2 \sigma_f \left(1 + \frac{d'}{d} + \frac{t_f}{2d} - 0.42\frac{x_u}{d}\right)$$

(Eq. 8)

and

$$\frac{M_u}{bd^2} = \frac{0.87 f_y p_t}{100}\left(1 - 0.42\frac{x_u}{d}\right)$$

(Eq.9) $$+ \frac{t_f}{d}\sigma_f\left(1 + \frac{d'}{d} + \frac{t_f}{2d} - 0.42\frac{x_u}{d}\right)$$

From above equations of moment it is clear that moment depends on the stress of fiber, thickness of fiber in addition to other usual values as in the conventional RC sections.

V. Preparation of Design Charts

A computer code is developed employing above mathematical formulation for the analysis of FRP strengthened RC beams. Thereafter, design charts are prepared by using the developed code for rectangular RC beams retrofitted with FRP to facilitate the designers, design charts have been proposed for rectangular beam retrofitted with the FRP sheets on the bottom portion of the RC beams using Eqs. 7 and 9. For retrofitting works, it is necessary to know the thickness of FRP plate needed to enhance the load carrying capacity. Design charts are to be made showing the different curves for different values of FRP plate thickness with known values of other variables such as steel percentage, width, depth, and cover of beam. On the other hand, for the purpose of new design or strengthening, it is necessary to know the amount of steel for a particular thickness of FRP plates or vice-versa. Hence twelve design charts are prepared taking different values of characteristics strength of concrete and different values of cover to depth ratio and presented in Figs 2-13. In this case we have presented the charts only for Fe 415. For other values the chart can be prepared accordingly**.**

VI. Results and Discussions

Since the results of strengthened RC beams retrofitted with FRP are not available in the literature, the accuracy of the present formulation is established by comparing the results of the RC beams without FRP available in the literature (SP:16-1978) with the results obtained from the present formulation.

Two rectangular beam of M_{20} and M_{30} Grade of concrete respectively and Fe 415 Grade steel with percentage of reinforcement is considered from SP:16-1978 to compare the moment of resistance in terms of (M_u/bd^2) obtained in MPa unit from the present formulation with that obtained from SP:16 –1978. The results are obtained as a special case considering the thickness of FRP plate is zero and presented in Table 1. It is observed that there is a good agreement between both the results.

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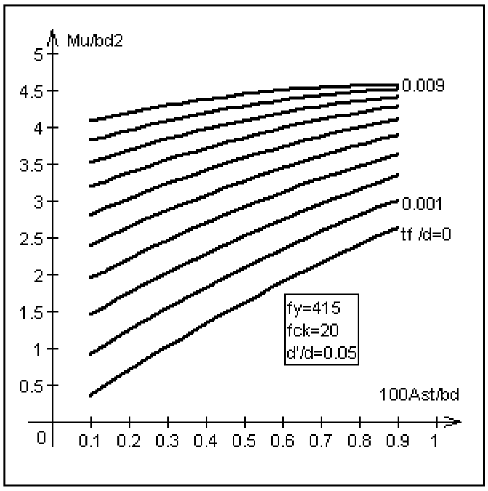

Fig. 2: Design chart 1

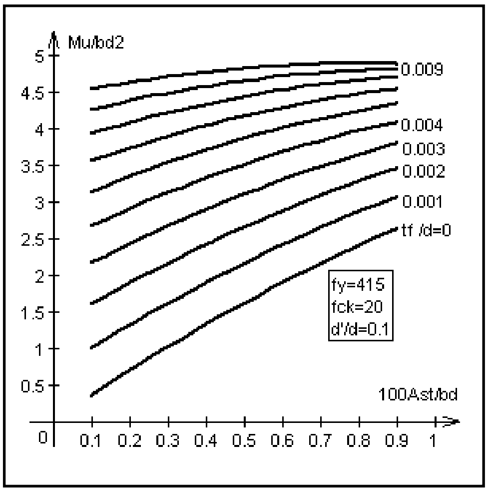

Fig. 3: Design chart 2

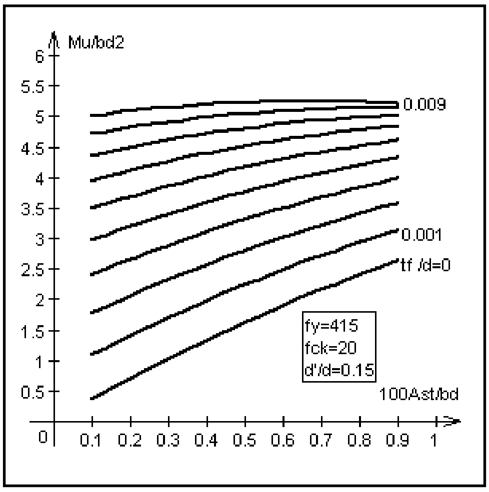

Fig. 4: Design chart 3

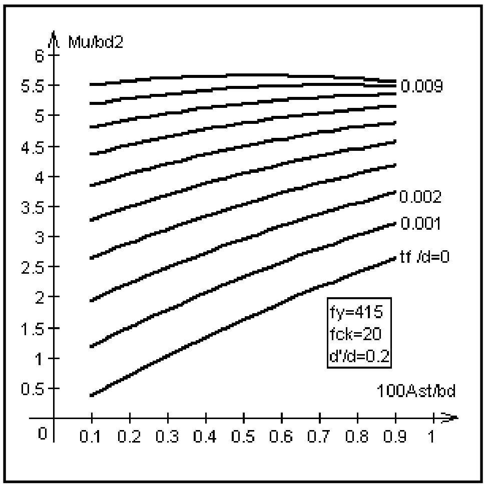

Fig. 5: Design chart 4

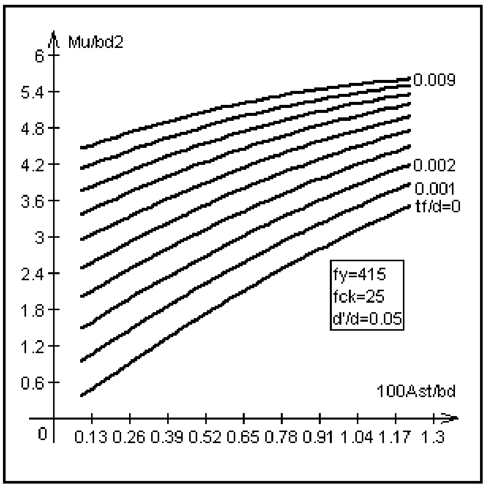

Fig. 6: Design chart 5

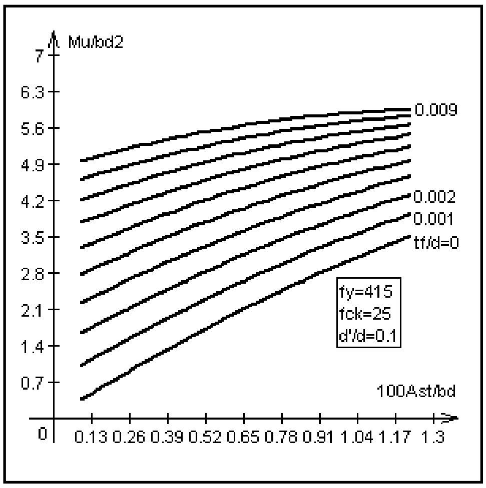

Fig. 7: Design chart 6

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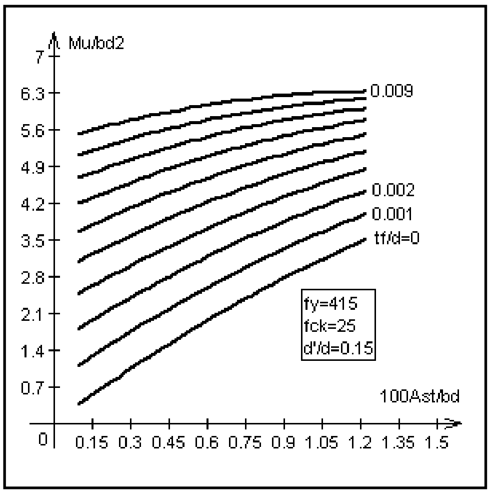

Fig. 8: Design chart 7

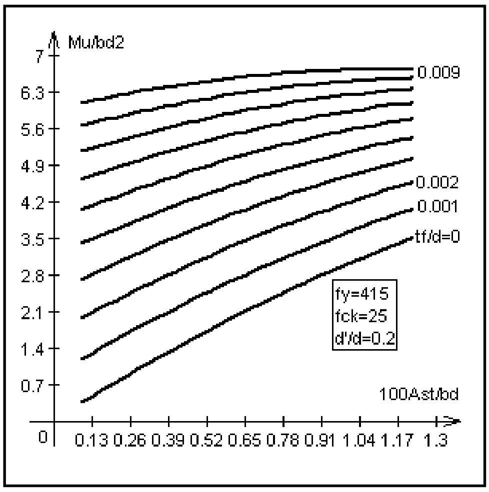

Fig. 9: Design chart 8

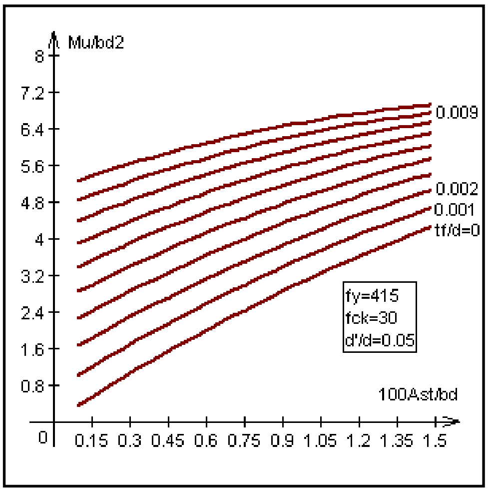

Fig. 10: Design chart 9

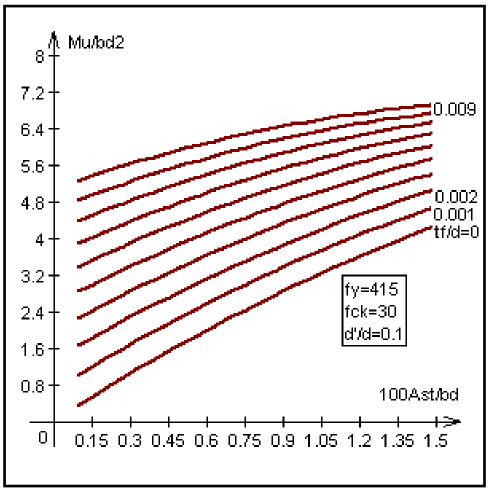

Fig. 11: Design chart 10

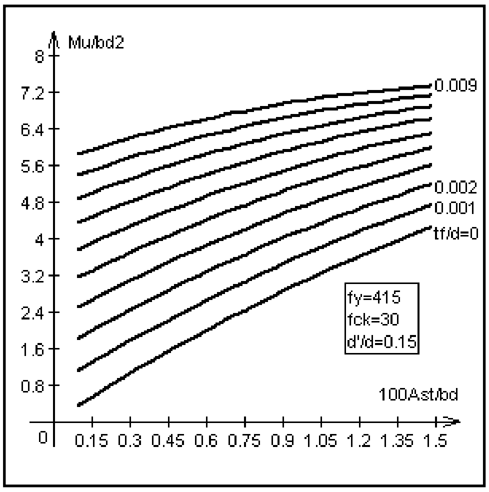

Fig. 12: Design chart 11

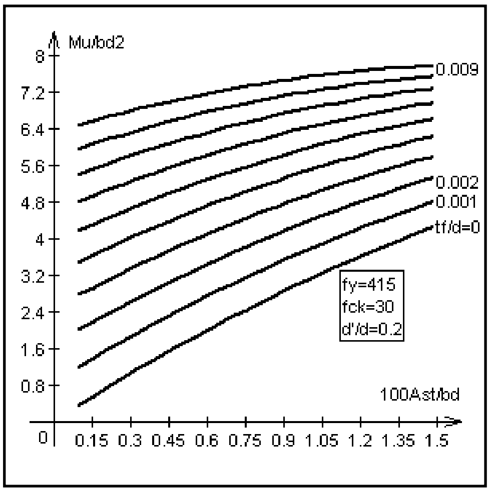

Fig. 13: Design chart 12

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TABLE I
COMPARISON BETWEEN VALUES OBTAINED FROM SP:16-1978 [ ] AND THOSE FROM PRESENT CHARTS

Sl. No.	f_{ck} (MPa)	f_y (MPa)	P_t	M_u/bd^2 (MPa)		Devi-ation (%)
				Ref.	Present charts	
1	20	415	0.114	0.40	0.4017	-0.425
2	30	415	0.156	0.55	0.5509	-0.164

Thereafter, ten different beams are taken with varying Grade of steel and varying percentage of steel. The cover to depth ratio and FRP thickness to depth ratio are also varied. For each case the Mu/bd^2 is calculated from the computer code developed and from proposed design charts and presented in Table 2 with percentage of deviation. The values of modulus of elasticity for steel and FRP are taken as 200 GPa and 140 Gpa, respectively. The value of f_y of steel is taken as 415 MPa for all the ten problems.From the Fig. 2, it is observed that the results, i.e., (M_u/bd^2) obtained from the code and from the proposed design charts are in good agreement. Hence, the designer can safely use the proposed design charts for estimating the flexural strength of RC beams strengthen with FRP sheets/laminates.

TABLE 2
COMPARISON BETWEEN VALUES OBTAINED FROM COMPUTER CODE AND THOSE FROM PRESENT CHARTS

Sl. No.	f_{ck} (MPa)	d′/d	t_f/d	P_t	M_u/bd^2 (MPa)		Devi-ation (%)
					Com-puter Code	Propo-sed charts	
1	20	0.05	0.001	0.25	1.387	1.390	-0.216
2	20	0.10	0.002	0.52	2.699	2.705	-0.222
3	20	0.15	0.003	0.70	3.674	3.670	+0.109
4	25	0.05	0.004	0.39	3.217	3.216	+0.031
5	25	0.10	0.005	0.65	4.432	4.438	-0.135
6	25	0.15	0.006	0.90	5.472	5.474	-0.036
7	25	0.20	0.007	1.04	6.266	6.265	+0.016
8	30	0.10	0.005	0.48	5.620	5.624	-0.071
9	30	0.15	0.006	0.96	4.660	4.664	-0.086
10	30	0.20	0.009	1.44	7.756	7.760	-0.052

VII. CONCLUSION

The following conclusions are drawn from the present study:

1. The paper presents mathematical formulation with extension of IS:456-2000 in order to estimate the flexural strength of RC beams retrofitted with FRP.
2. The formulation developed for the design of rectangular reinforced concrete beams strengthened with FRP plates, in accordance IS:456-2000,yields satisfactorily results.
3. The design charts with non dimensional parameters can be used effectively both for retrofitting the existing rectangular beams as well as for the design of new beams with FRP plates.

However, the above analytical model can be enhanced adopting different safety factors to FRP as in other national guide lines.

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Dr. A. N. Nayak, working as professor in the Department of Civil Engineering of VSS University of Technology, Burla, Odisha, has published more than 50 research papers in national and international conferences and journals of repute. He is also reviewer of many national and international journals including ASCE and Elsevier Publications. His research area of interest is retrofitting of concrete structures with FRP, structural dynamics and plates & shells.

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